Homework 6 Solutions
Physics 1000
Preparatory Physics
Due: Friday February 28, 1997

Reading Assignment

Basic Technical Physics, by Tippens. Chapter 6.

Written Assignment

Conceptual Questions

1. Yes or No. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration?

2. Which is greater, the acceleration of a motor cycle from 80 km/h to 90 km/h or the acceleration of a bicycle from rest to 10 km/h?

    (a) The motor cycle,

    (b) The bicycle,

    (c) Both the same,

    (d) You cannot tell from the information given.

3. Yes or No. Can the velocity of an object be negative when its acceleration is positive?

4. Yes or No. Can the velocity of an object be positive when its acceleration is negative?

5. A rock is thrown vertically upward with an initial speed from the edge of a cliff. A second rock is thrown vertically downward with the same initial speed. Which rock has the greater speed when it reaches the bottom of the cliff? Ignore air resistance.

    (a) The first rock.

    (b) The second rock.

    (c) Their speeds are the same.

    (d) You cannot tell from the information given.

Falling Bodies (Neglect air resistance in all problems)

6. Suppose a ball is dropped from a tower 70 m high. How far will it have fallen after:

    a. 1s: (4.9 m),

    b. 2s: (19.6 m),

    c. 3s?: (44.1 m)

    Answer: As always I choose a coordinate system at the base of the tower with positive up. $x_0 = 70$ m. The ball is dropped so $v_0 = 0$. Thus, distance fallen $= x_0 - x = gt^2/2$, by Galileo's fourth kinematical equation. Thus, for 1s distance fallen = g(1)/2 = 4.9 m, for 2s distance fallen = g(4)/2 = 19.6m, and for 3s distance fallen = g(9)/2 = 44.1 m.

7. What is the height of the ball mentioned in problem #6 above the ground after
   a. 1s: (65.1 m),
   b. 2s: (50.4 m),
   c. 3s?: (25.9 m)

   Answer: Height is just initial height, 70 m, minus distance fallen. So height at 1s is 70m - 4.9m = 65.1 m, height at 2s is 70m - 19.6m = 50.4 m, height at 2s is 70m - 44.1m = 25.9 m.

8. How long does it take the ball in problem #6 to hit the ground after it is dropped?

   Answer: The ball hits the ground when $x = 0$. Initial velocity is still $v_0 = 0$, so again by Galileo's fourth kinematical equation:
   $0 = x = x_0 - gt^2/2 \rightarrow t = \sqrt{2x_0/g} = \sqrt{140/9.8} = 3.8$ s.

9. Suppose the ball in problem #6 is thrown straight downward with a speed of 3 m/s, instead of being dropped. What then would be its height above the ground after:
   a. 1s: (62.1 m),
   b. 2s: (44.4 m),
   c. 3s?: (16.9 m)

   Answer: Now $v_0 \neq 0$, it is $v_0 = -3$ m/s. Height $x = x_0 + v_0 t - gt^2/2 = 70 - 3t - gt^2/2$. Thus, at 1s, $x = 70 - 3(1) - 4.9(1) = 62.1$m, at 2s, $x = 70 - 3(2) - 4.9(4) = 44.4$m, and at 3s, $x = 70 - 3(3) - 4.9(9) = 16.9$m.

10. What would be the speed of the ball in problem #9 be after:
    a. 1s: (-12.9 m/s),
    b. 2s?: (-25.6 m/s)

    Answer: Here we use Galileo's 2nd kinematical equation with $v_0 = -3$ m/s: $v = v_0 - gt$. At 1 s, $v = -3 - 9.8(1) = -12.9$ m/s and at 2 s, $v = -3 - 9.8(2) = -25.6$ m/s.

11. How long does it take the ball in problem #9 to hit the ground after it is thrown?

    Answer: To hit the ground, $x = 0$, and here $v_0 = -3$ m/s. These values need to be substituted into Galileo's 4th Kinematical Equation: $0 = x = x_0 + v_0 t - gt^2/2$. Rearranging yields:

$$t^2 - \frac{2v_0}{g}t - \frac{2x_0}{g} = 0. \tag{1}$$

As shown in class, the quadratic formula with $B = -2v_0/g$ and $C = 2x_0/g$ yields after some algebra:

$$t = \frac{v_0}{g} \pm \sqrt{\frac{v_0^2}{g^2} + \frac{2x_0}{g}}. \tag{2}$$

Substituting $v_0 = -3\text{m/s}$, $g = 9.8\ \text{m/s}^2$, and $x_0 = 70$ m into this equations yields $t = 3.48$ s or -4.1 s. The latter solution is physically nonsensical, so the solution is $t = 3.48$ s.

12. Suppose the ball in problem #6 is thrown straight upward (from the 70 m tower) into the air with an initial speed of 15 m/s.

    a. How high above the ground will it go?

Answer: At its highest point, $v = 0$. To do this problem we should use both Galileo's 2nd and 4th Kinematical equations:

$$\begin{aligned}
0 &= v = v_0 - gt \rightarrow t = v_0/g, && (3)\\
x &= x_0 + v_0 t - \frac{1}{2}gt^2 && (4)\\
&= x_0 + v_0\left(\frac{v_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0}{g}\right)^2 && (5)\\
&= x_0 + \frac{v_0^2}{2g} = 70 + \frac{(15)^2}{2(9.8)}m = 70 + 11.5m = 81.5m. && (6)
\end{aligned}$$

Note that in going from equation (4) to equation (5) we substituted from equation (3). The rest is algebra and substituting in the right numbers.

    b. How long will the ball be in the air before it hits the ground?

Answer: When the ball hits the ground $x = 0$, thus using Galileo's 4th kinematical equation again: $0 = x = x_0 + v_0 t - gt^2/2$. This is just the same equation used in problem #11 (eqn. (2)), where we showed the solution is:

$$\begin{aligned}
t &= \frac{v_0}{g} \pm \frac{1}{g}\sqrt{v_0^2 + 2x_0 g} && (7)\\
&= \frac{15}{9.8} \pm \frac{1}{9.8}\sqrt{225 + 1372}s && (8)\\
&= 1.53 \pm 4.1s = 5.6s \quad \text{or} \quad -2.6s. && (9)
\end{aligned}$$

As usual we drop the negative solution and we get the result that it will take 5.6 s for the ball to hit the ground.

Challenge Problem (Extra Credit)

A rock is dropped from a sea cliff and the sound of it striking the ocean is heard 3.4 s later. If the speed of sound is 340 m/s, how high is the cliff?

Answer: Since so many students did not do this problem I will make the solution brief. Let: h = height of the cliff, v_s = speed of sound = 340 m/s, t_{heard} = time to hear the echo = 3.4 s, t_{fall} = time it takes the rock to fall and strike the ocean, $t_{sound} = h/v_s$ = time it takes for sound to travel from the ocean surface to the top of the cliff. There are two equations we want to solve simultaneously:

$$t_{heard} = t_{fall} + t_{sound} = t_{fall} + \frac{h}{v_s} \quad (10)$$

$$h = x_0 - x = \frac{1}{2}gt^2. \quad (11)$$

Equation (10) in words merely states the obvious fact that the time for the sound to reach an observer at the top of the cliff is equal to the time it takes for the rock to fall plus the time it takes for the sound to travel from the ocean surface to the top of the cliff. Equaiton (11) is simply Galileo's 4th kinematical equation. Substituting from (10) into (11) and solving for h yields, after a few lines of algebra:

$$h^2 - 2\left(t_{heard}v_s + \frac{v_s^2}{g}\right)h + t_{heard}^2 v_s^2 = 0 \quad (12)$$

$$t_{heard}v_s + \frac{v_s^2}{g} = 3.4(340) + \frac{340^2}{9.8} = 12,952 \quad (13)$$

$$t_{heard}^2 v_s^2 = 1,336,336 \quad (14)$$

$$h^2 - 2(12,952)h + 1,336,336 = 0 \quad (15)$$

$$h = 12,952 \pm \sqrt{(12,952)^2 - 1,336,336} = 52m \text{ or } 25,852m. \quad (16)$$

Clearly, 25,852 m is much too large of an answer. So, the answer is that the cliff is 52 m high.