Homework 3
Physics 1000
Preparatory Physics
Due: Friday January 31, 1997

Reading Assignment

Basic Technical Physics, by Tippens. Chapter 1, sections 5-6; Chapter 2, sections 8,9; Ch. 3, sections, 1,2,5-7; Ch. 12, section 2.

Written Assignment

Volume, Mass, and Density (Tippens Ch. 2, sections 8 and 9; Ch. 12, section 2. Recall $\pi \sim 3.1416$.)

1. **Volume of a Right-Circular Cylinder.** What is the volume of a right-circular cylinder with radius 2 m and height 1 m? Remember to specify units. (See p. 47 of Tippens.)

   <u>Answer</u>: The volume of a right-circular cyliner is $\pi r^2 h$, where r is the radius and h is the height. $V = \pi r^2 h = (3.1416)(4)(1)$ m^3 = 12.6 m^3.

2. **Mass of a Mountain.** Consider a mountain with a volume of 1 km^3 (i.e., 1 cubic kilometer) and a density of 3 $\times 10^3$ kg/m^3. If mass is density times volume ($m = \rho V$), find the mass of the mountain.

   <u>Answer</u>: Mass = density $\times$ volume = ρV = (3 $\times 10^3$ kg/m^3)(1 km^3)$\left(\frac{10^3 m}{1km}\right)^3$= (3 $\times 10^3$ kg/m^3)(1 km^3)$\left(\frac{10^9 m^3}{1km^3}\right)$ = 3 $\times 10^{12}$ kg.

3. **Density.** Assume that the Earth is a sphere with a radius of 6,371 km and that Jupiter is also a sphere with a radius of Jupiter is 71,500 km. The Earth's mass is about 5.98 $\times 10^{24}$ kg and Jupiter's mass is about 1.90 $\times 10^{27}$ kg. Also, recall that if r is the radius of a sphere then the volume of that sphere is $4\pi r^3/3$. Finally, let us define the average density of a body, ρ, to be the body's total mass divided by its volume.

   a. What is the average density of the Earth, ρ_e? Show your work. Express your answer in scientific notation in kg/m^3.

   <u>Answer</u>: Average density = ρ = (Total Mass)/(Total Volume). We are told that the mass of the Earth is 5.98 $\times 10^{24}$ kg. Its volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.371 \times 10^3 \text{ km})^3 = \frac{4}{3}\pi(6.371)^3 \times (10^3)^3$ km^3 = 1,080 $\times 10^9$ km^3= 1.08 $\times 10^{12}$ km^3. So the average density is: ρ = (Total Mass)/(Total Volume) = (5.98 $\times 10^{24}$ kg)/(1.08 $\times 10^{12}$ km^3) = 5.54 $\times 10^{12}$ kg/km^3. The kg/km^3 is not the SI unit for density. If you have this answer, it's OK, however. But, the SI unit should be kg/m^3. Thus, ρ = 5.54 $\times 10^{12}$ kg/km^3 $(1km/10^3 m)^3$ = 5.54 $\times 10^{12}$ kg/km^3 $(1km^3/10^9 m^3)$ = 5.54 $\times 10^3$ kg/m^3.

   b. What is the average density of Jupiter, ρ_j? Show your work. Express your answer in scientific notation in kg/m^3.

   <u>Answer</u>: You could go through the same process as in (a), but in this problem you would simply substitute the mass and radius of Jupiter. It's somewhat easier doing the following, however. Let m_J = mass of Jupiter, m_E = mass of Earth, r_J = radius of Jupiter, r_E = radius of earth, V_J = volume of Jupiter, V_E = volume of Earth, ρ_J = density of Jupiter, and finally ρ_E = density of Earth. Note the following:

$$\begin{aligned} m_J &= 3.18 \times 10^2 m_E & (1) \\ r_J &= 11.2 r_E & (2) \\ V_J &= \frac{4}{3}\pi r_J^3 = \frac{4}{3}\pi(11.2r_E)^3 = (1.40 \times 10^3)\frac{4}{3}\pi(r_E)^3 = 1.40 \times 10^3 V_E & (3) \\ \rho_J &= \frac{m_J}{V_J} = \frac{3.18 \times 10^2 m_E}{1.40 \times 10^3 V_E} = 0.23\frac{m_E}{V_E} = .23\rho_E = 0.23(5.54 \times 10^3 kg/m^3) & (4) \\ &= 1.26 \times 10^3 kg/m^3. & (5) \end{aligned}$$

c. Why do you suppose the densities of these two planets are so different?

<u>Answer</u>: Jupiter is a 'gas-giant' composed mainly of hydrogen and helium and other gases. Earth is a rocky planet composed mainly of silicon and many other heavier elements (e.g., aluminum, iron, magnesium, oxygen, nitrogen, etc.). The Earth's density being greater than Jupiter's reflects the fact that it composed of heavier elements than Jupiter.

Quadratic Equation

4. **Review:** Recall that a quadratic equation is one that can be written as follows:

$$Ax^2 + Bx + C = 0, \tag{6}$$

where x is the variable we wish to solve for and A, B, C are parameters that will be fixed in the applications of this equation. There are two solutions for x, which are simply:

$$x = -\frac{B}{2A} \pm \frac{\sqrt{B^2 - 4AC}}{2A}. \tag{7}$$

For example, say we want to solve the equation $t^2 - 3t + 2 = 0$ for t. Comparing this equation with equation (1) we see that $A \to 1, B \to -3, C \to 2$. Note that in equation (1), x is the variable we want to solve for, but in our problem here, t is the variable we desire and which plays the same role in our problem as x does in eqution (1). Thus, equation (2) tells us that:

$$t = -\frac{-3}{(2)(1)} \pm \frac{\sqrt{(-3)^2 - 4(1)(2)}}{(2)(1)} = \frac{3}{2} \pm \frac{\sqrt{9-8}}{2} = \frac{3}{2} \pm \frac{1}{2}, \tag{8}$$

so $t = 2$ or $t = 1$.

The Problems:

a. Solve $x^2 - 7x + 10 = 0$ for x. **<u>Answer</u>:** $A = 1, B = -7, C = 10 \to x = \frac{7}{2} \pm \frac{3}{2} = 5$ or 2.

b. Solve $t^2 + 3t - 10 = 0$ for t. **<u>Answer</u>:** $A = 1, B = 3, C = -10 \to t = \frac{-3}{2} \pm \frac{7}{2} = -5$ or 2.

c. Solve $4t^2 + 6t - 10 = 0$ for t. **<u>Answer</u>:** $A = 4, B = 6, C = -10 \to t = \frac{-3}{4} \pm \frac{7}{4} = 1$ or -5/2.

Graphing (Tippens Ch. 1, section 5. Note: Tippens does not provide all of the information on graphing that we will need in this class. Added information is provided in Lecture 4 and subsequent lectures.)

5. **Equation of a Line → Graph, Slope, Intercept.** Using an $x-y$ grid in which y is the vertical axis and x is the horizontal axis. Label the positive x axis from 0 to 10 and the positive y axis from 0 to 11. (You can use one of the grids on the last page of the homework assignment. Label it clearly.)

   a. Graph $y = x + 1$ for x between 0 and 10. **Answer:** See the top left graph on the last page of these solutions.

   b. What is the value of the slope of this curve? **Answer:** $m = 1$.

   c. What is the value of the y-intercept of this curve? **Answer:** $b = 1$

6. **Points → Equation of a Line.** For each of the following pairs of points, find the equation that represents the straight line through the points. (Hint: To do this you use the general equation of a line, $y = mx + b$, where m is the slope and b is the y-intercept. Thus, this problem reduces to finding the slope and the y-intercept for the lines through each of the pairs of points below.)

   a. The points $(x, y) = (0, 1)$ and $(1, 2)$. (The point $(x, y) = (0, 1)$ is the point at which $x = 0$ and $y = 1$ and the point $(x, y) = (1, 2)$ is the point at which $x = 1$ and $y = 2$.)

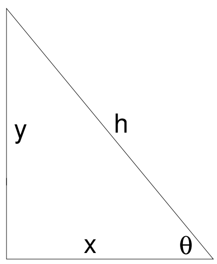
   Answer: slope $= m =$ Rise/Run $= (2 - 1)/(1 - 0) = 1$. y-intercept = b = the value of y when x is equal to $0 = 1$. Thus, $y = mx + b = x + 1$.

   b. The points $(x, y) = (0, -5)$ and $(1, -3)$. **Answer:** $m = (-3-(-5))/(1-0) = 2, b = -5$, $\mathbf{y = mx + b = 2x - 5}$.

   c. The points $(x, y) = (0, 1)$ and $(1, -4)$. **Answer:** $m = (-4 - (1))/(1 - 0) = -5, b = 1$, $\mathbf{y = mx + b = -5x + 1}$.

Right Triangles (Tippens Ch. 1, section 6; Ch. 3, section 5)

For the following two problems, consider the following picture of the right triangle labeled such that the horizontal side has length x, the vertical side has length y, the hypotenuse has length h, and the angle linking the horizontal side and the hypoteneuse is θ.



7. **Relating the Sides of a Right Triangle.** Which of the following are true?

$$\text{(a) } \sin\theta = \tfrac{y}{h} \quad \text{(c) } \tan\theta = \tfrac{y}{x}$$
$$\text{(b) } \cos\theta = \tfrac{y}{h} \quad \text{(d) } h = \sqrt{x^2 + y^2}$$

Answer: The answer is (a), (c), and (d).

8. **Using the Pythagorean Theorem.**

   a. If $x = \sqrt{7}$ and $y = 3$, what is h? **Answer:** $h = \sqrt{x^2 + y^2} = \sqrt{7+9} = \pm 4$. -4 can be ignored since the length of the side of a triangle is never negative. So, $h = 4$.

   b. If $y = 4$ and $h = 5$, what is x? **Answer:** $h = \sqrt{x^2 + y^2} \rightarrow h^2 = x^2 + y^2 \rightarrow x^2 = h^2 - y^2 \rightarrow x = \sqrt{h^2 - y^2} = \sqrt{25 - 16} = 3$, again ignoring the negative solution.

Vectors and Components of Motion (Tippens Ch. 3, section 6, 7)

9. Consider the motion of a car that goes 10 blocks north, 20 blocks East, and then 10 blocks north again.

   a. Plot the motion of the car as three vectors (arrows) with North as the y-direction and East and the x-direction: The first vector should have its tail at the origin pointing to 10 blocks north, the next from that location to 20 blocks east, and the last from that location to 10 blocks further north. Draw a fourth vector representing the total motion, with its tail at the origin ending at the final location. You can use one of the grids on the last page of the homework assignment. Label it clearly and make sure that your vectors do not end up off the graph. **Answer:** See the upper right plot on the last page of these solutions.

   b. What is the length (or magnitude) of the total displacement vector? **Answer:** $|\vec{\mathbf{x}}| = \sqrt{20^2 + 20^2}$ blocks $= 28.3$ blocks.

10. The following equation respresents a position vector, $\vec{x}$, in two dimensions:

$$\vec{x} = 2\hat{x} + 5\hat{y}. \tag{9}$$

Here, $\hat{x}$ and $\hat{y}$ are unit vectors in the x and y directions, respectively. Recall that a unit vector is a vector with unit length in the specified direction. Thus, they can be thought of specifying direction without modifying length. (Note: Tippens does not use unit vectors. These are discussed in Lecture 5 and subsequent lectures.)

   a. Using an $x - y$ grid, plot the vector with the tail at the origin. (You can use one of the grids on the last page of the homework assignment. Label it clearly.) **Answer:** See the lower left plot on the last page of these solutions.

   b. What is the length of this vector? **Answer:** $|\vec{\mathbf{x}}| = \sqrt{2^2 + 5^2} = \sqrt{29}$.