Homework 11
Physics 1000
Preparatory Physics
Due: Friday April 25, 1997

1. A child throws a rock horizontally off a 50 m cliff. The rock's initial velocity is 10 m/s. (Neglect air resistance.)

   a. How long will the rock be in the air?

   Answer: Note that the equations you use here are simply those that we derived for the ball falling from a ship's mast. $\Delta y = -gt^2/2 \rightarrow t = \sqrt{2y/g} = \sqrt{2(50)/9.8}$ s $= 3.2$s.

   b. How far from the base of the hill will the rock land?

   Answer: $x = v_0 t = (10)3.2$ m $= 32$ m.

2. Consider the monkey and the hunter problem. A hunter stands 10 m from the base of a tree in which a monkey is hanging 10 m above the ground and aims his gun directly at the monkey. The hunter shoots a water balloon at the monkey from ground level with an initial velocity of $\vec{v}_o = v_{xo}\hat{x} + v_{yo}\hat{y}$, where $v_{xo} = v_{yo} = 10$ m/s. The monkey drops from the tree the instant the balloon leaves the water-balloon-gun. (Neglect air resistance.)

   a. What is take-off angle of the water balloon relative to the horizontal?

   Answer: You are considering a right triangle in which the two sides other than the hypoteneuse are the distance between hunter and the tree and the height of the monkey in the tree. Both of these sides are equal to 10 m. So you have a right triangle with two equal sides, so the angle is just 45 degrees.

   b. What mathematical function describes the trajectory of the balloon while it is in the air?

   Answer: An object falling under the influence of gravity with a non-zero constant velocity in the horiztonal direction traces an arc which is simply a parabola.

   c. At what time after the balloon is fired will the balloon hit the monkey?

   Answer: The balloon will hit the monkey when $t = x/v_0 = (10/10)$ s $= 1$ s.

   d. Does the balloon hit the monkey on its way up or down? State your reasoning quantitatively. (Hint: Find the time it would take the balloon to reach its maximum height and see if that's before or after the monkey has been hit by the balloon.)

   Answer: The balloon continues to travel upward until it reaches its maximum height. At its maximum height, the y-component of the velocity of the balloon will 0, so the time to its maximum height can be found by considering the following kinematical equation: $0 = v_y = v_{0y} - gt = -gt$ since $v_{0y} = 0$ m/s. Thus, $t = v_{0y}/g = 10/9.8$ s $= 1.02$ s. So, the maximum height would be reached after the time the monkey was hit. Thus, the balloon was on the way up when it hit the monkey.

3. Consider two students, student A and student B, sitting 1 m apart in a lecture hall. Assume they both weigh 100 kg. What is the gravitational force between the students? State the magnitude and the direction. (Recall that G = 6.67 $\times 10^{-11}$ $m^3 kg^{-1} s^{-1}$ = 6.67 $\times$ 10^{-11} Nm^2/kg^2.)

<u>Answer:</u> $F = Gm_A m_B/r^2 = (6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2})\ (100\ \mathrm{kg})\ (100\ \mathrm{kg})/(1\mathrm{m})^2 = 6.67 \times 10^{-7}$ N.

4. The mass of the Earth is $m_e = 5.98 \times 10^{24}$ kg and the mass of the Sun is $m_s = 1.99 \times 10^{30}$ kg. The centers of mass of the Earth and Sun lie on average about 1.5 $\times 10^{11}$ m apart.

a. What is the magnitude of the gravitational force of attraction between these two objects? Express the answer in Newtons.

<u>Answer:</u> Let $r = 1.5 \times 10^{11}$ m. Then $F = Gm_e m_s/r^2 = (6.67 \times 10^{-11})(5.98 \times 10^{24})(1.99 \times 10^{30})/(1.5 \times 10^{11})^2 = (6.67)(5.98)(1.99)/(1.5^2) \times 10^{(-11+24+30-22)}$ N $= 35.3 \times 10^{21}$ N $= 3.53 \times 10^{22}$ N.

b. Using F=ma, what is the magnitude of the acceleration of the Earth towards the Sun caused by this gravitational force?

<u>Answer:</u> $a = F/m_e = (3.53 \times 10^{22}\ \mathrm{N})/(5.98 \times 10^{24}\ \mathrm{kg}) = (3.53/5.98) \times 10^{-2}$ $\mathrm{m/s^2} = 5.9 \times 10^{-3}$ $\mathrm{m/s^2}$.

5. Assume that the mass of the Earth is concentrated at its center. Consider a 100 kg person standing on the Earth's surface 6.371 $\times$ 10^6 m from the center of the Earth.

a. What is the magnitude of the gravitational force in Newtons that the Earth's mass (concentrated at its center) exerts on the person.

<u>Answer:</u> Let m be the mass of the person and $r = 6.371 \times 10^6$ m. $F = Gm_e m/r^2 = (6.67\times10^{-11})(5.98\times10^{24})(10^2)/(6.371\times10^6)^2 = (6.67)(5.98)/(6.371)^2 \times 10^{-11+24+2-12}$ N $= 0.98\times10^3$ N = 980 N.

b. Using F=ma, what is the magnitude of the acceleration of person towards the center of the Earth caused by this gravitational force?

<u>Answer:</u> $F = ma \rightarrow a = F/m = (980\ \mathrm{N})/(100\ \mathrm{kg}) = 9.8\ \mathrm{m/s^2}$.

c. How does this value compare with $g \sim 9.8$ $\mathrm{m/s^2}$.

<u>Answer:</u> They're about the same. Newton's Law of Universal Gravitation predicts the acceleration of gravity very accurately. More usefully, from a measurement of the acceleration of gravity, one can estimate the mass of the Earth.

d. In what direction does this acceleration point?

<u>Answer:</u> This acceleration points towards the Earth's center.

e. Using F=ma, what is the magnitude of the acceleration of center of the Earth towards the person caused by this gravitational force?

<u>Answer:</u> $F = ma_e \rightarrow a = F/m_e = (980\ \mathrm{N})/(5.98 \times 10^{24}\ \mathrm{kg}) = (9.8)/5.98 \times 10^{(-24+2)}$ $\mathrm{m/s^2} = .6 \times 10^{-22}$ $\mathrm{m/s^2}$. The Earth accelerates in response toward the person, but much more slowly!

f. In what direction does this acceleration point?

<u>Answer:</u> This acceleration points towards the person.