Homework 10 Solutions
Physics 1000
Preparatory Physics
Due: Friday April 11, 1997

1. A 10 kg object hangs from a vertical rope. Ignore the mass of the rope.

   a. What is the tension in the rope if the object is stationary?

   Answer: Coordinate system: + up, - down. If the object is stationary there is a balance of forces so the net force is zero: $0 = F_{net} = ma_y = T - mg \rightarrow T = mg = (10kg)(9.8\text{m/s}^2) = 98$ N.

   b. What is the tension in the rope if the object is moving upward with a constant velocity of 1 m/s?

   Answer: If the object is moving at a constant velocity it is still not accelerating and there is no net force acting on it, so the answer is the same as in (a): $T =$ 98 N.

   c. What is the tension in the rope if the object is moving downward with a constant velocity of 1 m/s?

   Answer: Same as (b).

   d. What is the tension in the rope if the object is moving upward with a constant acceleration of 1 m/s^2?

   Answer: $ma_y = T - mg \rightarrow T = m(a_y + g) =$ 10 kg (1 m/s^2 + 9.8/s^2) = 108 N.

   e. What is the tension in the rope if the object is moving downward with a constant acceleration of 1 m/s^2?

   Answer: $ma_y = T - mg \rightarrow T = m(a_y + g) =$ 10 kg (-1 m/s^2 + 9.8/s^2) = 88 N.

2. Two boxes are suspended by a single rope over a frictionless pulley. Box 1 weighs 10 kg and Box 2 weighs 12 kg. The only force operating on the boxes are friction and the tension in the rope. Ignore the mass of the pulley and the mass of the rope.

   a. What is the accleration of Box 1?

   Answer: Coordinate system: Mass 1 + up, Mass 2 + down. $m_1 = 10$ kg, $m_2 =$ 12 kg. $a_y = g(m_2 - m_1)/(m_2 + m_1) =$ [9.8(12-10)/22] m/s^2 = .89 m/s^2. This is up.

   b. What is the accleration of Box 2?

   Answer: Mathematically this is the same as in (a), $a_y =$ .89 m/s^2. The coordinate system has been chosen to insure this. However, for Mass 2 a positive acceleration is down.

   c. What is the tension in the rope?

   Answer: $m_1 a_y = T - m_1 g \rightarrow T = m_1(a_y + g) =$ 10(.89+9.8) N = 107 N.

3. Tow boxes are connected by a rope running over a pulley on the edge of a table. Box 1 is 5 kg and sits on a table. Box 2 is 2 kg and dangles from the rope attached to Box 1 which runs over the pulley. Ignore the mass of the rope and the pulley. The coefficient of friction between box 1 and the table is 0.2.

   a. What is the acceleration of Box 1?

   Answer: Coordinate system: Mass 1 + is down, Mass 2 + is to the right. Remember, to use the equation derived in class for the acceleration of this system, Mass 1 is dangling and Mass 2 is on the table. Thus, Mass 1 is Box 2 and Mass 2 is Box 1 here. (This is an unfortunate confusion that I did not intend.) Hence, $m_1 = 2$ kg, $m_2 = 5$ kg. $a_x = g(m_1 - \mu m_2)/(m_1 + m_2) = [9.8(2 - .2(5))/(2+5)]$ m/s^2 $= 1.4$ m/s^2. This is down.

   b. What is the acceleration of Box 2?

   Answer: Mathematically the same as in (a), The coordinate system has been chosend to guarantee this, but in this case the direction is to the right.

4. Consider a ball dropped from the mast of a moving boat. The boat is moving along the water at 5 m/s and the mast is 10 m over the surface of the water. Ignore air resistance.

   In class, when we considered this problem we found the following:

$$\begin{aligned} v_x &= v_0 & (1) \\ x &= x_0 + v_0 t & (2) \\ v_y &= -gt \quad \text{and} \quad |v_y| = \sqrt{v_{y0}^2 - 2g(y - y_0)} & (3) \\ y &= y_0 - \frac{1}{2}gt^2 & (4) \end{aligned}$$

   Here, $v_{x0} = v_0 = 5$ m/s. The coordinate system is chosen with the origin at the water level at the initial location of the boat so $x_0 = 0$, $y_0 = 10$ m, $v_{y0} = 0$ m/s, and $y = 0$ when the ball hits the water. Substituting these values into equations (1) - (4) yields:

$$\begin{aligned} v_x &= 5m/s & (5) \\ x &= (5m/s)t & (6) \\ v_y &= -gt \quad \text{and} \quad |v_y| = \sqrt{2g(10m)} & (7) \\ y &= 10m - \frac{1}{2}gt^2. & (8) \end{aligned}$$

   a. What is the x-component (horizontal) of the velocity of the ball when it hits the water?

   Answer: By equation (5), v_x is constant and equal to 5 m/s.

   b. What is the y-component (vertical) of the velocity of the ball when it hits the water?

   Answer: By equation (7), $|v_y| = \sqrt{2(9.8)(10)}$ m/s $= 14$ m/s. The ball is moving downward, so v_y should be negative: $v_y = -14$ m/s.

c. How will the ball be in the air before it hits the water?

Answer: Unfortunately, this question contains a typo. It is meant to ask 'How LONG will the ball be in the air.....'. No points should be deducted. To answer the question, use equation (7) and the result from (b), to find that $t = -v_y/g = (14/9.8)$ s $= 1.43$ s.

d. How far will the ball travel horizontally while it is in the air?

Answer: Use equation (6) and the time from (c): $x = (5$ m/s) t $= (5)(1.43$ s) $= 7.2$ m.

e. What is the final speed of the ball while it's in the air? (Hint: To answer this question use the definition that speed is the magnitude of the velocity vector. You need to use the Pythagorean theorem and both the x- and y-components of velocity to find speed.)

Answer: speed $= \sqrt{v_x^2 + v_y^2} = \sqrt{5^2 + (-14)^2}$ m/s $= 14.9$ m/s.